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ELECTRIC OSCILLATIONS.

Les Oscillations Électriques. Par H. Poincaré, Membre de l'Institut, Rédigées par M. Ch. Maurain. (Paris: Georges Carré, 1894.)

WE have already noticed at some length M. Poincaré's works on *Électricité et Optique*, and given an account of his views regarding Maxwell's theories, and his mode of presenting the subject in the light of Hertz's researches. The present work discusses more particularly the subject of Electric Oscillations, and includes comparatively little of the more abstract treatment of electrical theory to be found in the others. We have in it a very instructive and well-timed *résumé* of a good deal of the later work on the subject, with mathematical investigations of many points arising in connection with the various experimental researches referred to, which are of great value.

In a short preliminary statement of the theory underlying the Hertzian experiments, M. Poincaré adopts, to a certain extent, the mode of presentation used by Hertz himself. The two reciprocal sets of equations also made by Heaviside the starting-point of his researches, and called by him the circuital equations, are made by Hertz the basis of everything. Thus if L, M, N, X, Y, Z be the components of the magnetic and electric forces, μ and ϵ the "coefficients of magnetic and electric induction," and A the reciprocal of the velocity of light, the equations are

$$A\mu \left(\frac{\partial L}{\partial t}, \frac{\partial M}{\partial t}, \frac{\partial N}{\partial t} \right) = \left(\frac{\partial Z}{\partial y} - \frac{\partial Y}{\partial z}, \frac{\partial X}{\partial z} - \frac{\partial Z}{\partial x}, \frac{\partial Y}{\partial x} - \frac{\partial X}{\partial y} \right),$$

$$A \left(\epsilon \frac{\partial X}{\partial t} + 4\pi u, \text{ \&c.} \right) = \left(\frac{\partial M}{\partial z} - \frac{\partial N}{\partial y}, \text{ \&c.} \right)$$

where u, v, w are the components of conduction current at x, y, z . When $u, v, w = 0$, one set of equations can thus be formed from the other set by interchange of L, M, N with X, Y, Z , and μ with $-\epsilon$. The electric energy is $\epsilon(X^2 + Y^2 + Z^2)/8\pi$, and the magnetic energy $\mu(L^2 + M^2 + N^2)/8\pi$, each taken per unit of volume of the medium.

The units of force are so chosen that μ and ϵ are both unity for vacuum, that is, for ether. Hence the introduction of A in the equations above. It is required to make the dimensions of both sides alike. We must confess that we much prefer the idea contained, though not clearly expressed, in Maxwell's treatise, of regarding μ and K as quantities dependent on the physical properties of the medium, and such that the velocity of light in the medium is $1/\sqrt{\mu K}$. There can be no question of the enormous advantage of thus proceeding. It avoids the obvious absurdity of giving in a general system of measurement founded on the units of length, mass, and time, two different sets of dimensions to the same quantity according as it is measured electrostatically or electromagnetically. Further, if this were recognised, the serious difficulty which besets students, when they find that what is defined or explained as a mere ratio is unity, or the reciprocal of the square of the velocity of light, according to the system of units adopted, would entirely disappear. The discussion which took

place in 1882 regarding the dimensions of magnetic pole in electrostatic units, gave illustrations of the confusion and misconception to which neglect of the dimensional relation between μ and K can give rise in the mind of even a master of physical science.

Let μ_0, K_0 , or better, μ_0, κ_0 , be the inductive capacities (magnetic and electric) of ether, μ, κ , those of any other substance, then $\mu/\mu_0, \kappa/\kappa_0$ will be the magnetic and electric permeabilities of the medium, and will be mere ratios. The magnetic permeability will then be as Lord Kelvin originally defined it, the ratio of two magnetic forces, of the magnetic force defined electromagnetically (in a crevasse across the direction of magnetisation) to the magnetic force according to the polar definition (in a narrow cylindrical hollow parallel to the magnetisation), and our ideas will no longer be liable to obfuscation in an important fundamental matter.

M. Poincaré derives the second set of equations stated above from the first set and the principle of conservation of energy, using for the total energy per unit of volume the sum of the expressions quoted above. The theoretical basis consists thus of (1) the experimental fact that the line-integral of electric force round a circuit is equal to the time-rate of variation of the total magnetic induction through the circuit, which gives the first set of equations; (2) the expressions for the magnetic and electric energies; and (3) the theorem that the work spent in heat per unit of volume per unit of time is $Xu + Yv + Zw$. The expressions for the energies are thus taken as fundamental. The result does not, however, depend on any supposition as to exact localisation of the energy; all that is involved is the expression of the energy as an integral extended through the whole field.

This is not the same as the process of Hertz, and it is not clear that it possesses any advantage over that method. In fact, Hertz's foundation is the two sets of reciprocal equations, which are first postulated, then shown to be consistent with observed phenomena. Thus the whole structure rests on the equations, and Hertz is but little, if at all, concerned with the dynamical explanation of electromagnetic phenomena. This, on the other hand, Maxwell is continually; and one of the most remarkable chapters of his book is that in which he applies the method of Lagrange to give a dynamical basis to the theory of induction and electromagnetic action. This was a natural outcome of the other process, which he followed, of establishing his equations as far as possible directly from the experimental facts, and connecting them by a strong web of dynamical theory. It does not seem likely, moreover, that the minds of many of those who are eagerly seeking for some more intimate knowledge of electromagnetic action will be content with anything but a dynamical explanation of electrical phenomena, and one which shall elucidate, in some measure at least, the relation of the ether to ordinary matter, and the reason for the existence of the ponderomotive forces exerted in the electromagnetic field.

In chapter iii. M. Poincaré proceeds to the theoretical study of Hertzian oscillations, and discusses in the first instance the solution of the differential equations for the case of conductors situated in a dielectric. The first part of this does not call for special remark. X is taken

in the ordinary way as the real part of $(B - iC)e^{ipt}$ where $i = \sqrt{-1}$, and $p = q - i\gamma$. The value of p is here misprinted $\gamma + iq$. The author then proceeds to find solutions for the case of an indefinitely extended dielectric, and throwing the equations into the form

$$L, M, N = \frac{\partial \xi}{\partial y} - \frac{\partial \eta}{\partial z}, \text{ \&c.,}$$

that is introducing the vector-potential, he goes on to solve the resulting equations for X, Y, Z . Two solutions are obtained of the form

$$\xi = -A \int \frac{u}{r} dx dy dz$$

where (u) may have either of the values u', u'' , given by $u' = f(x', y', z', t)$, $u'' = f(x', y', z', t - Ar)$. Here u' is the value of the x -component of the current at the point (x', y', z') at time t , u'' the value of the same component at the same point at a time previous by the interval required for the light to travel from (x, y, z) to (x', y', z') , r being the distance between these two points. The latter solution is, of course, adapted to the case of propagation in space with velocity $1/A$. It is exemplified by application to the complete solution of Lodge's spherical vibrator. This, in the assumed absence of radiation of energy and frictional damping out of the electrical vibration, may be regarded, for points external to the sphere, as simply a time-periodic electric doublet; for its electrification may be imagined produced by the relative displacement through a small distance of two uniform spherical volume distributions, each of which acts as if its whole charge were situated at its centre. But the existence of radiation very materially affects the result, for if r be the distance of any point from the centre of the vibrator, a the radius of the sphere, ψ a function which gives the electric forces by the equations

$$X = \partial^2 \psi / \partial x \partial z, \quad Y = \partial^2 \psi / \partial y \partial z, \\ Z = -A^2 \partial^2 \psi / \partial t^2 + \partial^2 \psi / \partial z^2,$$

the direction of the axis of the vibrator being that of z then

$$r\psi = Be^{\frac{r}{2a}} e^{-t/2aA} \cos \frac{\sqrt{3}}{2a} \left(r - \frac{t}{A} \right) \\ + Ce^{\frac{r}{2a}} e^{-t/2aA} \sin \frac{\sqrt{3}}{2a} \left(r - \frac{t}{A} \right).$$

This expression is interesting as showing exactly the damping due to radiation in this case; that due to frictional generation of heat is neglected. As $1/A$ is the velocity of light, it will be seen that the radiation damping is exceedingly rapid. This illustrates the difficulty of "maintaining" electric oscillations; and has an important bearing on the interpretation of results obtained with resonators which are damped slowly as compared with the exciters.

Of course, the damping in complicated cases can be found when the total flow of energy through a spherical surface of large radius, surrounding the vibrator, can be computed by Poynting's theorem. Thus it is possible to construct a more accurate solution for points distant from the vibrator, than that given by the supposition of no damping, by thus calculating approximately the exponential factor, and we can go on by successive approximation if need be.

The results obtained by Hert with regard to the period and the damping out of the vibrations of his exciter were

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tested, of course, by his resonator; and we have, in chapter iv. of M. Poincaré's work, an excellent account of the mathematics of resonance, so far as here required, and of the propagation of electric waves along wires, illustrated by reference to the experiments of Sarasin and De la Rive, Blondlot, and others. Many of these have reference to the variation of the current with distance from the end of the wire, at which, of course, reflections took place. The theory, for example, of Mr. D. E. Jones' experiments, in which a thermo-electric couple was used to measure the heating effect at different distances from the end of a long wire, is worked out; and the results, when compared with those of the experiments, are found to agree.

Chapters follow on Propagation in Air, and Applications of Theory. In the latter, M. Poincaré refers, among other things, to the results of the very important experiments of Bjerknes, on waves received by resonators of different metals; that is, their different rates of damping, and the depths to which the currents penetrate into the metal as measured by the thickness (to take an example) of copper, which must be deposited on an iron resonator in order that it may behave like one of solid copper.

These three chapters have a very important bearing on the question of multiple resonance, to which M. Poincaré has himself devoted special attention, and form, perhaps, the most valuable part of the work.

Lastly, the book deals with such applications of electrical oscillations as the determination of specific inductive capacities, the reflection and refraction of electric waves, and ends with a study of Hertz's "Memoir on the Fundamental Equations of Electrodynamics for Moving Bodies."

The mathematical discussions are, as we have indicated, throughout lucid and very frequently elegant. The facility with which an apparently intractable problem is taken hold of, and a few more or less elementary considerations made to yield an approximate solution or result, is very remarkable. We could have wished for a somewhat more physical treatment of the subject; but to those whose physical ideas have had some training with regard to these matters, M. Poincaré's treatise cannot but be of great service, as supplying what no doubt was its object, a review of the principal results obtained in this field of research compared with theory, as far as possible by the only sure test, that of calculation.

A. GRAY.

THE BOOK OF THE ROSE.

The Book of the Rose. By the Rev. A. Foster-Melliard, M.A., Rector of Sproughton, Suffolk. Pp. 336. 29 illustrations. (London: Macmillan and Co., 1894.)

MR. MELLIARD is well known among horticulturists as a successful grower and exhibitor of roses, and his book is what he wished it to be—the rose considered as a flower, with full details for its practical culture for amateurs, from the beginning to the end. Art is his text all through. He has very little to say about the botany of the rose, its geographical distribution, the origin of the numerous races of garden roses as distinguished from their wild progenitors, the fourteen chapters of his book